$$x=\frac{ac\pm\sqrt{(a^2c^2+240abcn)}}{2bc}.$$

Substituting the proposed values for a, b, c, and n gives, $x=8$, and $x-a/b=7\frac{1}{2}$.

Also solved by *G. B. M. ZERR.*

GEOMETRY.

153. Proposed by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.

If P, P', Q, Q' be the extremities of two chords of a conic section, and both chords pass through the point A, show that the sum of the squares of the reciprocals of AP, AP', AQ, AQ' is constant.

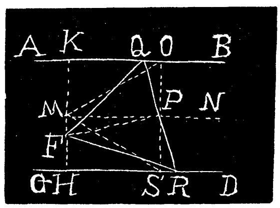
No solution of this problem has been received.

156. Proposed by F. M. McGAW, A. M., Professor or Mathematics, Bordentown Military Institute, Bordentown, N. J.

To construct an equilateral triangle such that its vertices shall be in each of two parallel lines and a point fixed between these lines.

Solution by G. I. HOPKINS, A. M., Professor of Mathematics and Physics, High School, Manchester, N. H.

Let AB and CD be the two parallel lines, and F the fixed point between them. Through F draw HK perpendicular to CD. Make $\angle NMO=30°$. Draw MN the perpendicular bisector of HK. Draw OS perpendicular to CD. Join F and P, and through P draw QR perpendicular to FP. Join QF and RF, then FQR is the required triangle.



PROOF. Triangles QOP and MFP are right triangles. $\angle QPO=\angle MPF$, being complements of the same $\angle QPM$.

$\therefore$ these triangles are similar. $\therefore$ $OP:MP::QP:FP$, or by alternation $OP:QP::MP:FP$. But these are homologous sides of the triangles OPM and QPF also.

$\therefore$ these triangles are similar, since they are right triangles and the legs proportional. But the $\angle OMP$ is 30° and $\angle MOP$ is 60°.

$\therefore$ $\angle QFP$ is 30° and $\angle FQP$ is 60°. Triangle FPR is easily shown to be equal to triangle FPQ.

$\therefore$ $\angle FRP=60°$. $\therefore$ triangle FQR is equiangular and therefore equilateral.

Excellent solutions were received from G. M. M. Zerr, H. C. Whitaker, J. Scheffer, and Theodore Linquist. Professors Zerr's and Whitaker's solutions were by analytical geometry; Professor Scheffer's solution was by trigonometry and the application of algebra to geometry; and Professor Linquist, of the Kansas Agricultural College, gave a very good construction by pure geometry.

157. Proposed by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.

Find the locus of the center of a circle touching a given line and always passing through a given point.

I. Solution by J. SCHEFFER, A. M., Hagerstown, Md.; ELMER SCHUYLER, M. Sc., Reading, Pa.; LON C. WALKER, Palo Alto, Cal.; H. C. WHITAKER, Ph. D., Philadelphia, Pa.; and the PROPOSER.

Take the given line as the axis of x, the line through the given point and at right angles to the given line as the y-axis, and denote the given point as $(0, y_1)$.

The required circle being of the form $x^2+y^2+2gx+2fy+c=0\ldots.(1)$, and touching $y=0\ldots.(2)$, $x^2+2gx+c=0\ldots.(3)$, and $c=g^2\ldots.(4)$.

Also, passing through $(0, y_1)$, $y_1^2+2fy_1+c=0\ldots.(5)$, and this with (4) gives $2f=-\dfrac{g^2+y_1^2}{y_1}\ldots.(6)$.

(1) now is $x^2+y^2+2gx-\dfrac{g^2+y_1^2}{y_1}y+g^2=0\ldots.(7)$.

If (x', y') be the center of (7), $x'=-g$, $y'=\dfrac{g^2+y_1^2}{2y_1}$, and eliminating g from these two equations, $x'^2=2y_1(y'-\frac{1}{2}y_1)$, a common parabola.

II. Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.; P. S. BERG, B. Sc., Larimore, N. D.; and H. R. HIGLEY, East Stroudsburg, Pa.

Since the distance of the center from the given straight line is always equal to its distance from the given point, both being equal to the radius of the circle, the locus of the center is a parabola having the given straight line for directrix and the given point for focus.

ANOTHER PROOF OF THE PYTHAGOREAN THEOREM.

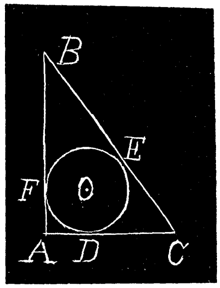
By E. S. LOOMIS, Ph. D., Teacher of Mathematics, West High School, Cleveland, Ohio.

Let ABC be a right triangle whose sides are tangent to the circle O. Since $CD=CF$, $BF=BE$, and $AE=AD=r=$radius of circle, it is easily shown that $(CB=a)+2r=(AC+AB=b+c)$. And if $a+2r=b+c\ldots.(1)$, then $(1)^2=(2)$ $a^2+4ar+4r^2=b^2+2bc+c^2$. Now if $4ar+4r^2=2bc$, then $a^2=b^2+c^2$. But $4ar+4r^2$ is greater than, equal to, or less than $2bc$.

If $4ar+4r^2>$ or $<2bc$, then $a^2+4ar+4r^2>$ or $<b^2+2bc+c^2$; *i. e.* $a+2r<$ or $>b+c$, which is absurd.

$\therefore$ $4ar+4r^2=2bc$.

$\therefore$ $a^2=b^2+c^2$. Q. E. D.



NOTE. So far as we know, this proof has not been given before. If it has not been published before, it may be properly called a *new proof*. Dr. Loomis asks if any one can derive, by this method, a direct proof—the one above being indirect. ED. F.

CALCULUS.

116. Proposed by JOHN M. COLAW, A. M., Monterey, Va.

"Prove that the length of the *greatest* beam of square section that can be cut from a log l feet long and in the shape of a conic frustum, diameters D and d, is $\frac{2}{3}lD\div(D-d)$ feet.'